Mixed-sensitivity design of a dynamic controller for systems pre-compensated by input shapers

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Abstract: We employ recent control design algorithms for delay systems to address the mixed sensitivity design of a dynamic controller for multiple degree of freedom systems interconnected with an inverse signal shaper. A delay based inverse shaper is included within a feedback loop in order to eliminate oscillatory modes due to an attached flexible substructure, which has been shown to be advantageous to inducing directly the filtering property by an appropriate controller design. Due to the presence of the inverse shaper, the closed loop dynamics become infinite dimensional. Furthermore we design fixed-order controllers, aiming at an easy implementation. As a consequence of these two features standard techniques from H-infinity control can no longer be used. We present a two-stage method, which firstly makes the closed-loop system asymptotically stable and, secondly, computes a low-order controller that minimizes the H-infinity norm of the weighted closed-loop transfer functions. The approach is grounded in recent methods for fixed-order H-infinity control of delay systems, and the delay-differential algebraic equations (DDAE) modelling and control framework.

Keywords: time-delay; spectral optimization; H-infinity control; inverse shapers;

1. INTRODUCTION

The use of feedforward control to compensate undesired oscillations is a widely used technique, which was firstly proposed in 1957 by Smith and well-know as the posicast filter Smith (1957). As shown in Fig 1, the reference command $w(t)$ for the nominal system P_0 controlled by a feedback controller K is shaped by the input shaper S in such a way that the target oscillatory mode of the attached flexible structure F is not excited. Subsequently, the research focused on more robust versions, was introduced in Singhose et al. (1994). Extensions to multi-mode shapers able to eliminate two or more modes simultaneously were proposed in Sung and Singhose (2009); Tuttle and Seering (1994), and the corresponding discrete time versions in Cole (2011).

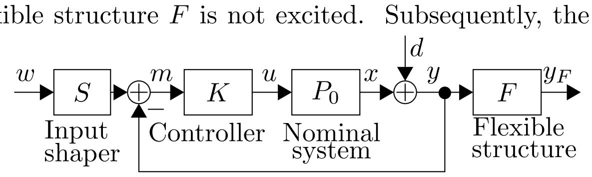

Fig. 1. Classical feedforward application of input shaper.

Recently, a novel architecture where an inverse shaper is included in a feedback loop, has been proposed in Vyhlídal et al. (2015). This architecture provides filtering of the undesired frequency for the reference input but also for input and output disturbances. This properties are not achievable by a standalone controller. For more details on inverse shaper and their relation to other shaping techniques we refer to Vyhlidal et al. (2013); Vyhlídal et al. (2015); Vyhlídal and Hromčík (2016). However, as shown in Pilbauer et al. (2016), the controller can be designed to mimic properties of the inverse shaper but only for specific inputs and not all of them. Additionally, the controller has some other constraints which limit its usage. For more details see Pilbauer et al. (2016).

By inversion of the shaper and the feedback interconnection, the shaper's zeros are turned into poles, and the overall closed loop would be a delay system of neutral type, characterized by an undesirable spectrum location and a sensitivity of stability with respect to infinitesimal delay perturbations, and none of the classical shapers with lumped delay are applicable in this context. Instead, new types of shapers based on distributed delays need to be applied, Vyhlídal et al. (2013a,b); Pilbauer et al.

(2015a); Vyhlídal and Hromčík (2015), providing retarded dynamics of the closed loop system.

The crucial task for the systems with inverse shaper in the feedback is the controller design. Even though the dynamics of both the system and the controller are considered as delay free and finite-dimensional, the overall closed-loop dynamics become infinite dimensional due to inclusion of the inverse shaper containing delays. The design is however simplified by the fact that the delays are fixed and known. As pointed out in Vyhlídal et al. (2015), the classical frequency-response design methods are convenient for the given task. Based on the analysis of conceptually simpler single degree of freedom application in Vyhlídal et al. (2015), it has been demonstrated that as long as the controller is designed as sufficiently fast and as long as it features good gain and phase margins in combination with the system, inclusion of the inverse shaper dynamics in the closed loop does not bring critical closed-loop stability concerns.

In this paper, next to addressing a more complex multiple degrees of freedom (MDOF) systems, a robust control design method is proposed for the system-inverse shaper interconnection. It is based on minimizing the H-infinity norm of the closed loop weighted mixed sensitivity functions. Let us remark that it is a subsequent work to Pilbauer et al. (2015b), where a static feedback controller was considered for the given task. In this paper, more complex dynamic feedback controller is considered providing us more flexibility in the loop shaping in order to increase robustness against external noise and model uncertainties. Besides, conceptually different method based on minimizing the spectral abscissa was used in Pilbauer et al. (2015b), which can easily handle the requirement on the prompt actions of the controller, but lacks the ability to predefine a requirement on a robust controller setting.

The mixed-sensitivity control design approach was proposed in Zames (1981). A direct of application to the problem under consideration is not possible for two reasons. First, due to the presence of the shaper, the closed-loop system is an infinite-dimensional time-delay system. Second, as we aim to design finite-dimensional controllers, which are easy to implement, the design problem can be seen of a reduced order control design problem, known to be difficult to solve. In view of these properties, we employ the algorithm and software for fixed-order H-infinity synthesis proposed in Gumussoy and Michiels (2011) and Gumussoy and Overton (2008).

Since the algorithm of Gumussoy and Michiels (2011) relies on an asymptotically stabilizing initial controller, our design consists of two stages. Firstly, we design a stabilizing controller by the method minimizing the spectral abscissa, as proposed in Michiels (2011); Vandewalle et al. (2008) Once the stabilizing controller is obtained, the H-infinity norm criterion, stemming from the mixed-sensitivity problem formulation, is optimized.

As a by product of the overall control design procedure we illustrate the power of the delay differential algebraic equations (DDAE) modeling framework: the system components (uncontrolled system, inverse shaper, controller), connections between components, and the weights for the H-infinity design, can be directly combined in one system of DDAEs, which are supported by both the algorithm of Michiels (2011) and the one of Gumussoy and Michiels (2011).

2. PRELIMINARIES

A closed loop architecture with the inverse shaper proposed in Vyhlídal et al. (2015), is shown in Fig. 2. Consider nominal system with an input u and the output x has the strictly proper transfer function $P_0(s)$, and Flexible structure features the transfer function $F(s)$, with y being the system output and $y = d + x$ being its input, d is unmeasurable disturbance. The feedback position controller for Nominal system has a transfer function $K(s)$. The inverse shaper transfer function is given by $\frac{1}{S(s)}$. The purpose

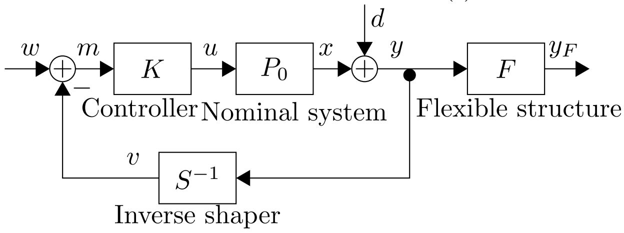

Fig. 2. Inverse shaper in the feedback architecture

of including the inverse shaper to the feedback loop is to project its filtering (mode compensation) properties, to the transfer functions of the closed loop system

$$T_{y_F w}(s) = \frac{K(s)P_0(s)}{1+K(s)P_0(s)\frac{1}{S(s)}}F(s) = \frac{K(s)P_0(s)S(s)}{S(s)+K(s)P_0(s)}F(s), \quad (1)$$

$$T_{y_F d}(s) = \frac{1}{1+K(s)P_0(s)\frac{1}{S(s)}}F(s) = \frac{S(s)}{S(s)+K(s)P_0(s)}F(s). \quad (2)$$

If the shaper $S(s)$ is designed to compensate poles $r_{1,2}$ of the flexible structure by its zero $s_{1,2} = r_{1,2}$, this zero-pole compensation will project to the transfer functions (1) and (2), because $S(s)$ appears in their numerators.

A general description of a delay based input shaper is as follows,

$$v(t) = ay(t) + (1-a)\int_0^{\vartheta} y(t-\mu)dh(\mu), \quad (3)$$

where y and v are the shaper input and output, respectively, $a \in \mathbb{R}^+, a < 1$ is the gain parameter, and the distribution of the delays is prescribed by the non-decreasing function $h(\mu)$, with length ϑ. Considering that overall delay consists of a series of lumped and equally distributed delay of the lengths τ, we obtain delayed-zero-vibration(DZV) shaper (see Vyhlídal et al. (2013a) for more details). Additionally, as shown in Vyhlídal and Hromčík (2015); Pilbauer et al. (2015a), selection of the delay distribution can improve robustness of the shaper against variations from the exact oscillatory mode. On the other hand, complexity of the robust shapers brings difficulty in controller design when the shaper is included in feedback. Therefore, DZV shaper is used to demonstrate the design of a dynamic controller. Transfer function of DZV shaper is in the form

$$S(s) = a + (1-a)\frac{1-e^{-sT}}{Ts}e^{-s\tau} \quad (4)$$

The time domain interpretation of the equation can be seen in Fig. 3(top). The bottom of Fig. 3 shows frequency response of the shaper, where frequency to suppress ($\omega_0 = 0.1 rad/s$) is observable as drop of the output amplitude in the ω_0 neighbourhood. Thus, the input shaper performs the task of a notch filter. Its key advantage compared to this classical filtrating technique is the monotonously increasing step response (or nonnegative impulse response). This results in an energetically convenient solution to perform the task. Performing the same task with a classical filter would be very difficult and has many limitations as described in Pilbauer et al. (2016).

The shaper is parameterized by the lengths of the distributed delay T,the lumped delay τ and by the gain a, as proposed in Vyhlídal et al. (2013b). From application

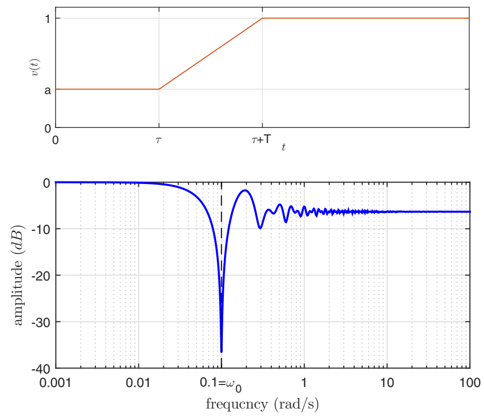

Fig. 3. top: step response of the shaper (4); bottom: magnitude frequency response of the shaper (4)

point of view, the inversion of the shaper is described by

$$v(t) = \frac{1}{a}\left(y(t) - (1-a)r(t)\right), \tag{5}$$

where

$$r(t) = \frac{1}{\tau}\int_{t-(T+\tau)}^{t-\tau} v(\eta)d\eta. \tag{6}$$

with the output v and the input $y = x_2 - d$. For the synthesis of the model and the controller design we assume disturbance $d = 0$. In our controller design procedure we implement equation (6) dynamically as

$$\dot{r}(t) = \frac{1}{T}(v(t-\tau) - v(t-(T+\tau))). \tag{7}$$

This transformation results in additional dynamics, characterized by the introduction of a non-physical eigenvalue at zero.

3. SYNTHESIS OF THE PROBLEM

3.1 Problem statement

Consider system P_0, shown in Fig. 2, which can be described by

$$P_0 \begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) + Du(t) \end{cases} \tag{8}$$

where $x(t) \in \mathbb{R}^n$ is the vector of system states, $u(t) \in \mathbb{R}$ being the control input and capital letters are real-valued matrices of appropriate dimensions. Apparently (see Fig. 2), the subsystem F is considered as decoupled from P_0. The additional information on the system F is its oscillatory mode given by the natural frequency ω_0 and the damping ratio ζ, determining the complex conjugate couple of poles $r_{1,2} = -\zeta\omega_0 - j\omega_0\sqrt{1-\zeta^2}$ which will be used for the shaper design. The inverse shaper within the feedback will filter the given modes from both the reference w and disturbances d. However, the inverse shaper introduces time delays into the closed loop system and therefore the system becomes infinite dimensional. If the system has undesired dynamics or is even not stable a controller is essential part of the scheme although design of the controller is more challenging. We firstly propose fixed-order controller which is lately designed with a two step approach, consisting of stabilization and robust H-infinity design for time-delay system.

3.2 Control objectives and associated optimization problem

The original system (8) without feedback is of finite dimension. By including the inverse shaper (3)-(7) in the feedback, the system becomes infinite dimensional and, therefore, infinitely many eigenvalues are introduced into the system. Considering that the nominal system (8) can be unstable, we propose a fixed-order dynamic feedback controller in order to stabilize and optimize the system in the form

$$K : \begin{cases} \dot{x}_K(t) &= A_K x_K(t) + B_K m(t), \\ u(t) &= C_K x_K(t) + D_K m(t), \end{cases} \tag{9}$$

where $x_K(t) \in \mathbb{R}^n$ is the vector of the controller states, $m(t) \in \mathbb{R}$ being the controller input and capital letters are real-valued matrices of appropriate dimensions.

As mentioned above, the design of the controller has multiple steps regarding to different control objectives.

Compensation of the flexible mode Compensation of the oscillatory modes of the flexible structure is automatically achieved by interconnection of the inverse shaper in the feedback with properly tuned parameters.

Stabilization As the H-infinity optimization requires system together with stabilizing controller, the system has to be stabilized. Stabilization is achieved by minimizing the spectral abscissa of the closed-loop, leading to the optimization problem

$$\min_K c(K), \tag{10}$$

where the spectral abscissa is defined as

$$c(K) := \sup\left\{\Re(s) : \ s \in \Sigma(K) \setminus \{0\}\right\}, \tag{11}$$

where the characteristic function of the closed loop $\Sigma(K)$ is function of the parameters of the fixed-order controller (9) as a function of the elements of matrices (A_K,B_K,C_K,D_K).

Note that the integral expression for r in (6) was turned into a differential equation with discrete delays, (7), by differentiation, which introduces a non-physical zero eigenvalue. In the eigenvalue computations this eigenvalue needs to be removed from the computed spectrum (11), which is done by removing the computed eigenvalue with smallest modulus.

H-infinity optimization The basic formulation comes from Fig. 4, consisting of generalized plant G with the stabilizing controller K. As shown in Fig. 5, the plant G consists of nominal system P_0 together with weighting filters and inverse shaper, K is the controller to be synthesised. Inputs w and u are exogenous input and control input, outputs z and y are exogenous outputs to be optimized and output to the controller, respectively. The

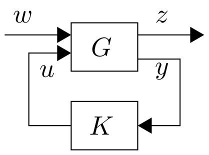

Fig. 4. Standard H-infinity control scheme

task is to design an output dynamical controller K (9) to achieve desired disturbance rejection, reference tracking performance and robustness against system uncertainties, which can be transformed into optimization problem

$$\alpha := \min_K \left\| \begin{matrix} W_1 Q(K) \\ W_2 T(K) \end{matrix} \right\|_{\mathcal{H}_\infty}, \tag{12}$$

where $Q = (1+P_0 S^{-1} K)^{-1}$ and $T = (1+P_0 S^{-1} K)^{-1} P_0 K$ are sensitivity and complementary sensitivity functions, respectively. Weighting filters W_1 and W_2 are designed in order to get required performance. Filter W_1 should be selected as low pass filter to give good tracking performance for low frequencies and limit overshoot for high frequencies. The filter W_2 is selected as high pass filter in order to achieve stability robustness against noise and unmodeled dynamics. Above that, the cut-off frequency of $\frac{\alpha}{W_1}$ should be selected higher than is the frequency of the given mode in order to obtain a system which is fast enough to track the trajectory generated by the inverse shaper. The second weighting filter $\frac{\alpha}{W_2}$ should have cut-off frequency selected with respect to a knowledge of the environment, used sensors and actuators which brings extra unmodeled dynamics and noise into the system.

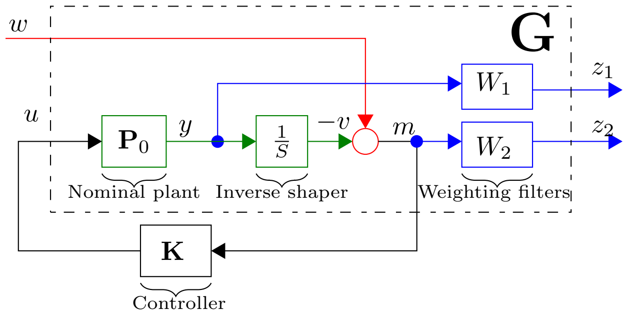

Fig. 5. Generalized plant with weighting filters

3.3 Reformulation in standard DDAE form

For the controller design we model the overall system using delay differential algebraic equations (DDAEs), as in Michiels (2011). As shown in Fig. 5, the system **G** also includes weighting filters W_1 and W_2, described in state-space representation as

$$W_1 \begin{cases} \dot{x}_{W_1} = A_{W_1} x_{W_1} + B_{W_1} y, \\ y_{W_1} = C_{W_1} x_{W_1} + D_{W_1} y, \end{cases} \tag{13}$$

$$W_2 \begin{cases} \dot{x}_{W_2} = A_{W_2} x_{W_2} + B_{W_2} m, \\ y_{W_2} = C_{W_2} x_{W_2} + D_{W_2} m. \end{cases} \tag{14}$$

Next, one algebraic equation for relation between reference input w and output of the inverse shaper v is needed

$$m = w - v. \tag{15}$$

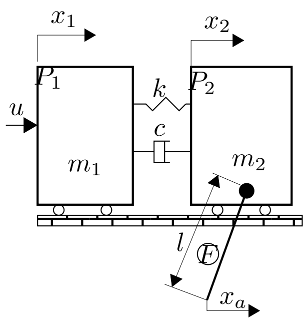

Fig. 6. Mechanical set-up consisting of two carts P_1-P_2 and a pendulum F as a flexible subsystem.

By putting together equations (8) for the nominal system P_0, (5) and (7) for the inverse shaper and for weighting filters equations (13)-(14), the following systems of differential and delay-difference equations is obtained:

$$G \begin{cases} \dot{x}(t) = Ax(t) + Bu(t), \\ \dot{r}(t) = \frac{1}{\tau}(v(t-\tau) - v(t-(T+\tau))), \\ 0 = -v(t) + \frac{1}{a}(y - (1-a)r(t)), \\ 0 = -m(t) + w(t) + v(t), \\ \dot{x}_{W_1}(t) = A_{W_1} x_{W_1}(t) + B_{W_1} v(t), \\ \dot{x}_{W_2}(t) = A_{W_2} x_{W_2}(t) + B_{W_2} m(t), \end{cases} \tag{16}$$

with new generalized state as
$X(t) = [x_1(t) \cdots x_n(t)\ \dot{x}_1(t) \cdots \dot{x}_n(t)\ r(t)\ v(t)\ m(t)\ x_{W_1}(t)\ x_{W_2}(t)]^T$.
The equations can be written in the form

$$\begin{cases} \mathcal{E}\dot{X}(t) &= \mathcal{A}_0 X(t) + \mathcal{A}_1 X(t-\tau) + \mathcal{A}_2 X(t-(T+\tau)) + \\ & \quad \mathcal{B}_u u(t) + \mathcal{B}_w w(t), \\ y(t) &= \mathcal{C}_y X(t), \\ z(t) &= \mathcal{C}_z X(t). \end{cases} \tag{17}$$

The system (16) is now in form (17) as standard DDAEs, which is used in software for stabilization Michiels (2011) and H-infinity optimization Gumussoy and Michiels (2011).

Before we apply the routines, some preceding modification needs to be done. As mentioned earlier, by including shaper (4) in the integral form (5)-(7) we introduce one non-physical zero eigenvalue. The eigenvalue has to be removed from the spectra during both the stabilization and optimization procedures.

4. CASE STUDY EXAMPLE

Consider the mechanical system from Fig. 6, consisting only of two sub-systems $\mathbf{P}_1$ and $\mathbf{P}_2$, which is described by

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{k}{m_1} & \frac{k}{m_1} & -\frac{c}{m_1} & \frac{c}{m_1} \\ \frac{k}{m_2} & -\frac{k}{m_2} & -\frac{c}{m_2} & -\frac{c}{m_2} \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{m_1} \\ 0 \end{bmatrix}, \tag{18}$$

where $x = [x_1, x_2, \dot{x}_1, \dot{x}_2]$, x_1, x_2 denoting displacements of the carts and u being the control input. The flexible structure $\mathbf{F}$ is considered as pendulum connected to the second subsystem $\mathbf{P}_2$. Next, as the subsystem $\mathbf{F}$ has much smaller mass then $\mathbf{P}_2$ $m_a << m_2$, the flexible part is considered as decoupled from $\mathbf{P}_1 - \mathbf{P}_2$.

The parameters of the system are the masses m_1 and m_2, damping c and stiffness k coefficients.

We demonstrate the proposed control scheme by providing the following simulation example. The parameters are considered as $m_1 = 1kg$, $m_2 = 1kg$, $k = 1000Nm^{-1}$, $c = 5kgs^{-1}$. The flexible structure oscillatory mode to be compensated is defined with natural frequency $\omega_0 = 0.1s^{-1}$ and damping ratio $\zeta = 0.01$. The input shaper (4) parameters to compensate the given oscillatory mode are determined as $a = 0.482$, $\tau = 0.47s$ and $T = 0.63s$. Order of the controller should be selected sufficiently high, here we used 4th order. The system (18) has only two unstable eigenvalues at the origin and the stabilization needs only few steps to obtain stabilizing controller. Since the key part of the controller design is H-infinity optimization, the minimization of spectral abscissa does not need to reach the minimum.

The weighting filters are selected in order to meet required dynamics of the system. The weighting function W_1 is selected to reflect the desired reference tracking performance and to limit overshoot, where both criteria are necessary to meet to have good filtering properties of the inverse shaper. The second filter W_2 is designed to perform better insensitivity to noise and unmodelled dynamics, which usually has most of its energy concentrated at high frequencies. Besides, the cut-off frequency for inversion of the filters $\frac{\alpha}{W_1}$ and $\frac{\alpha}{W_2}$ should be on a higher frequency than anti-resonant frequency ($\omega_0 = 0.1rad/s$) of the inverse shaper. Transfer functions of the weighting filters, meeting the defined criterion, were selected as

$$W_1 = \frac{0.5s + 0.4}{s + 0.002}, W_2 = \frac{0.25s + 0.6}{0.01s + 5}. \qquad (19)$$

Fig. 7 shows the result of the optimization problem (12) which reached the optimum $\alpha = 1.455$. As can nicely be seen on both the sensitivity curves characteristics, next to fulfilling the objectives raised by the robust control design, both the curves keep the notch filter characteristics injected to the system by the input shaper $S(s)$. This is the key property and requirement on the considered design technique. Next, we can see, that weighting function $\frac{\alpha}{W_1}$ is touching the sensitivity function Q in multiple places and both, the Q and T transfer functions, lie below weighing filters, which indicates that the optimization reached correct solution. Response of the system to the

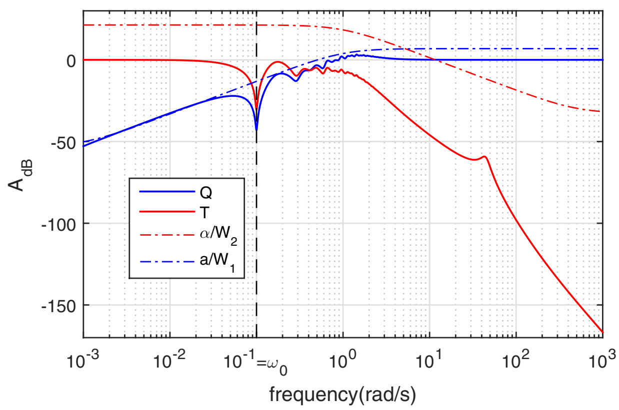

Fig. 7. Sensitivity(blue solid) and complementary sensitivity(red solid) function with the corresponding filters(dashed lines).

change of the reference signal is in Fig. 8. The imprint of the shaper to the dynamics of the closed loop can be seen in the upper chart. The projection of the time-distributed (time-delay) nature of the feedback shaper to the feedback system responses is the key benefit of the proposed control scheme. The almost non-decreasing character of the set-point response copying the shaper $S(s)$ response characteristics (compare Fig. 3(top) and Fig. 8) would be beneficial particularly in the task of positioning the mechanical systems. Such a valuable response shape would hardly be achieved if a classical notch filter was used instead of the input shaper. The position x_a of the flexible structure is in the bottom of the figure, demonstrating functionality of zero-pole cancellation provided by inverse shaper. The settling of the flexible structure is within one period of its nominal frequency. As the main benefit of

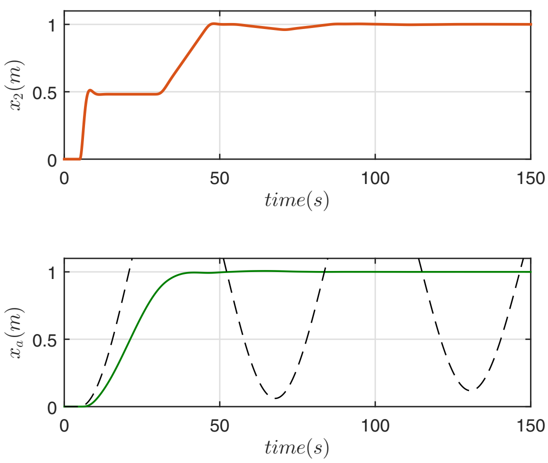

Fig. 8. Response of the system to change of reference $w = 1$ at time $t = 5$. Top: position of the second cart P_2; bottom: position of the endpoint of the pendulum F(green solid line) in comparison with system without shaper(black dashed line).

including the inverse shaper in the feedback is to introduce disturbance rejection, response to the output disturbance d is shown in Fig. 9, where disturbance d acts on the output of x_2. As can be seen, when the cart P_2 returns to the original position, there are no visible oscillations of the pendulum x_a.

5. CONCLUSIONS AND FUTURE WORKS

We demonstrated robust design of the dynamic controller for systems with inverse shaper within the system feedback. Due to inclusion of the delay based inverse shaper, standard methods for the design of the controller are not applicable and hereto we presented techniques tailored for the given tasks. The shaper is included in order not to excite oscillatory modes of low damped flexible structure for both the change of reference signal and external disturbance, with balanced energy utilization which would be difficult to achieve with classical filtrating techniques. On the other hand, when the inverse shaper with time delays is included, its dynamics projects in the closed loop and the spectrum becomes of infinite dimension and the system

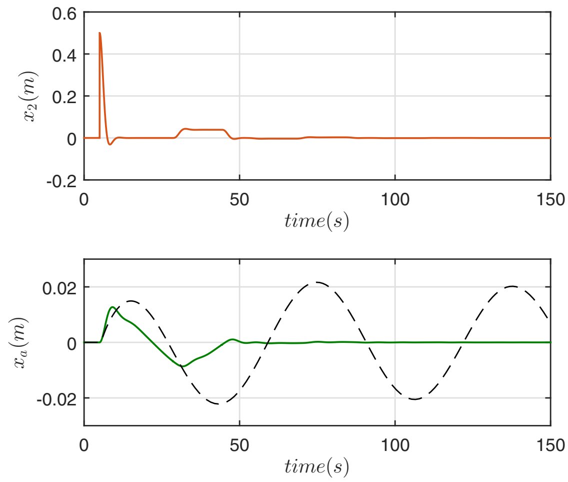

Fig. 9. Response of the system to output disturbance $d = 0.5$ at time $t = 5$. Top: position of the second cart P_2; bottom: position of the endpoint of the pendulum F in comparison with system without shaper(black dashed line).

requires different treatment. Next to the compensation of the oscillatory modes, the dynamical controller is used to stabilize the unstable poles of the system. The proposed dynamical controller provides more degrees of freedom for shaping the transfer functions as it may serve as an implicit observer. The stabilization is provided by minimizing of the spectral abscissa. Once the stable system is obtained, the final step is to shape system's transfer functions by optimizing the H-infinity norm. The H-infinity controller design is supported by weighting filters, which allows to define a shape of the transfer functions with regards to the required performance. In future work we will focus on systems with multiple inputs/outputs and perform the experimental verification of the proposed scheme. Next possible extension is to enhance the H-infinity approach to multi-objective optimization problem, which could enrich the shaping of the transfer functions.

6. ACKNOWLEDGMENTS

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REFERENCES

Cole, M.O. (2011). A discrete-time approach to impulse-based adaptive input shaping for motion control without residual vibration. *Automatica*, 47(11), 2504–2510.

Gumussoy, S. and Michiels, W. (2011). Fixed-order h-infinity control for interconnected systems using delay differential algebraic equations. *SIAM Journal on Control and Optimization*, 49(5), 2212–2238.

Gumussoy, S. and Overton, M.L. (2008). Fixed-order h controller design via hifoo, a specialized nonsmooth optimization package. In *Proceedings of the 2008 American Control Conference*, 2750–2754.

Michiels, W. (2011). Spectrum-based stability analysis and stabilisation of systems described by delay differential algebraic equations. *IET control theory & applications*, 5(16), 1829–1842.

Pilbauer, D., Michiels, W., and Vyhlídal, T. (2015a). Distributed delay input shaper design by optimizing smooth kernel functions. *TW Reports*, TW663. Submitted to The Journal of The Franklin Institute.

Pilbauer, D., Michiels, W., and Vyhlídal, T. (2016). A comparison of shaper-based and shaper-free architectures for feedforward compensation of flexible modes. *TW Reports*, TW672. Submitted to Advances in Delays and Dynamics- Delays and Interconnections: Methodology, Algorithms and Applications.

Pilbauer, D., Vyhlidal, T., and Michiels, W. (2015b). Spectral design of output feedback controllers for systems pre-compensated by input shapers. In *Proceedings of the 12th IFAC Workshop on Time-Delay Systems*, 1–6.

Singhose, W., Seering, W., and Singer, N. (1994). Residual vibration reduction using vector diagrams to generate shaped inputs. *Journal of Mechanical Design*, 116(2), 654–659.

Smith, O.J. (1957). Posicast control of damped oscillatory systems. *Proceedings of the IRE*, 45(9), 1249–1255.

Sung, Y.G. and Singhose, W. (2009). Robustness analysis of input shaping commands for two-mode flexible systems. *Control Theory & Applications, IET*, 3(6), 722–730.

Tuttle, T.D. and Seering, W.P. (1994). A zero-placement technique for designing shaped inputs to suppress multiple-mode vibration. In *American Control Conference, 1994*, volume 3, 2533–2537. IEEE.

Vandewalle, S., Michiels, W., Verheyden, K., and Vanbiervliet, J. (2008). A nonsmooth optimisation approach for the stabilisation of time-delay systems. *ESAIM: Control, Optimisation and Calculus of Variations*, 14(3), 478–493.

Vyhlídal, T. and Hromčík, M. (2015). Parameterization of zero vibration shapers with delays of various distribution. *Automatica*, 59, 256–263.

Vyhlídal, T. and Hromčík, M. (2016). Inverse feedback shapers for coupled multibody systems. *Submitted to IEEE Transactions on Automatic Control*.

Vyhlidal, T., Hromcik, M., and Kucera, V. (2013). Inverse signal shapers in effective feedback architecture. In *Control Conference (ECC), 2013 European*, 4418–4423. IEEE.

Vyhlídal, T., Hromčík, M., Kučera, V., and Anderle, M. (2015). On feedback architectures with zero vibration signal shapers. *IEEE Transactions on Automatic Control*, PP(99), 1–1. doi:10.1109/TAC.2015.2492502.

Vyhlídal, T., Kučera, V., and Hromčík, M. (2013a). Signal shaper with a distributed delay: Spectral analysis and design. *Automatica*, 49(11), 3484–3489.

Vyhlídal, T., Kučera, V., and Hromčík, M. (2013b). Zero vibration shapers with distributed delays of various types. In *Decision and Control (CDC), 2013 IEEE 52nd Annual Conference on*, 940–945. IEEE.

Zames, G. (1981). Feedback and optimal sensitivity: Model reference transformations, multiplicative seminorms, and approximate inverses. *Automatic Control, IEEE Transactions on*, 26(2), 301–320.